CHINESE JOURNAL OF PHYSICS VOL. 38, NO. 2-II APRIL 2000

Collective Excitations of a Composite Fermion Gas in Fractional Quantum Hall Effect

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(Received August 17, 1999)

The equations-of-motion method is used to calculate excitation energies of a composite-fermion gas in fractional quantum Hall effect. The gauge theory of composite fermions is applied in this study. Full gauge fluctuations, which contain two-body gauge interactions and three-body gauge interactions, are investigated. Comparing with excitations calculated by the random-phase approximation, collective-excitation spectra are changed dramatically by the three-body gauge interactions. In order that the gauge theory agrees with the finite-size studies, we find that the three-body gauge interactions are important in excitations of composite fermion gas. Both cases with and without Coulomb interactions are considered. Except the cyclotron energy, Coulomb interactions raise the gap energies at $q = 0$. Coulomb interactions have no effect on the cyclotron energy, in accordance with Kohn's theorem.

PACS. 73.40.Hm – Quantum Hall effect.
PACS. 73.20.Dx – Electron states in low-dimensional structures.
PACS. 73.20.Mf – Collective excitations.

I. Introduction

A two-dimensional (2D) charged particles in a strong magnetic field B perpendicular to the 2D plane has unique properties that arises from the quantization of the 2D continuum of kinetic energy states into a discrete series of Landau levels. The discreteness of the eigenvalue spectrum is essential for the occurrence of the quantum Hall effect. The integral quantum Hall effect is an effect of electron gas in a magnetic field. The fractional quantum Hall effect (FQHE) is an effect of composite-fermion (CF) gas in a magnetic field, where Coulomb interactions (CI) are responsible for the formation of composite fermion [1].

The CF picture asserts that a quantum Hall liquid of electrons in an external magnetic field B_v, corresponding to the Landau level filling of $v = p/(2mp+1)$ (m, p : integers), is equivalent to a liquid of composite fermions each carrying $\theta = 2m$ Chern-Simons (CS) gauge flux quanta. Composite fermions are immersed in the effective magnetic field $B_{eff} = B_v - B_{1/2m}$, which corresponds to the Landau level filling of $v = p$. The FQHE of electrons is understood as the integral quantum Hall effect of composite fermions. Electrons at $v = 1/2m$ are mapped to a CF gas in zero effective magnetic field.

Kamilla, Wu and Jain [2] developed techniques for computing the excitation spectrum using the Jastrow CF wave functions. The incompressible FQHE states are understood, as filled Landau levels of composite fermions, and the low-energy neutral excitations are excitons of composite fermions. The positions of various minima were explained by analogy to the integral quantum Hall effect. They obtained additional minima of excitation spectrum for $v = 1/3$ state, where one minimum was observed in the single mode approximation [3]. Additional minima arise simply from interplay between the structures in the density profiles of the quasielectron and quasihole, as the distance between them is varied. For $v = 1/3$ state, the ratio between the lowest energy gap at wave vector q=0 and the energy at the minimum is greater than 2. This suggests that the actual lowest-energy excitation at small q may contain two pairs of CF excitons forming a quadrupole [3, 4].

A CF gas in an effective magnetic field is responsible to the occurrence of the FQHE under the mean-field approximation. Lopez and Fradkin [5] proposed that, going beyond mean-field theory, one could employ the random-phase approximation (RPA) to study fluctuations from CS gauge field. The response functions and excitation spectra were calculated within the RPA in the long wavelength limit. They found a family of collective modes with dispersion relations whose zero-momentum gap is $k\omega_c$, where k is an integer number different from 1 and $2mp + 1$. Here $\omega_c = eB_{eff}/m_b c$ is the effective cyclotron frequency, where m_b is the bare electron band mass. When $k = 2mp + 1$, i.e., the zero-momentum gap is the cyclotron frequency, there is a splitting in the dispersion relation for finite wave vector q. This degenerate cyclotron mode can be viewed as the mixing of the modes with $k = 1$ and with $k = 2mp + 1$. CI, which are crucial for the occurrence of the FQHE, play only a nominal role in the CS approach. According to the RPA, the lowest collective-energy gap at $q = 0$ is not affected by CI. This phenomenon of the RPA indicates that CI have no effect on the incompressibility of the FQHE, which is certainly wrong. This problem is dealt phenomenological through a Landau Fermi-liquid theory and the effective mass is introduced as a free parameter [6].

The RPA, in its standard form, as used by Lopez and Fradkin, assumes an effective mass, which is equal to the bare electron band mass. If one arbitrarily changes the value of the mass in the RPA in order to get reasonable energies for the lowest branch of the excitation spectrum for the FQHE, then one violates both the f-sum rule and Kohn's theorem, which says that in the limit $q = 0$ a mode at the cyclotron frequency has all the weight of the f-sum rule. Simon and Halperin [6] did CS calculations in the modified RPA for the excitation spectra, which account for the effective-mass renormalization within a Landau-Fermi-Liquid theory approach. The modified RPA is expected to be accurate in the large Landau levels filled, where the motion of CF becomes semiclassical. While it provides a way of dealing with the problem of mass renormalization, its phenomenological nature is unsatisfied. For small Landau levels filled, the semiclassical approximation is expected to break down due to the strong effective magnetic field.

Although there have been several CF studies of the collective mode dispersion using the CS approach. There is still no satisfied CS theory to describe the CF gas correctly. Lack of satisfied theory is because previous studies do not treat gauge fluctuations properly. Three-body gauge interactions (3BGI) from gauge fluctuations were ignored by the RPA. A renormalized mass as a free parameter was considered to account 3BGI in the modified RPA implicitly. In order to treat gauge fluctuations properly, we believe that 3BGI should be included explicitly in any theory. Based on this fact, we propose a study of the effects from 3BGI acting on CF collective excitations. In this report we present the first parameter-free calculation of CF collective modes, where complete gauge fluctuations are considered. Complete gauge fluctuations contain both two-body gauge interactions and 3BGI.

There are many different ways to calculate the excitation spectrum of any system; the

Green's-functions method, the time-dependent Hartree-Fock theory, and the method of linearized equations of motion. It is very difficult to apply these methods on 3BGI. In this report we propose to use the equations-of-motion method, proposed by Rowe [7] and used in this report, is particular simply one which shows much greater flexibility than the linearized equation of motion. It has the following form:

$$\langle\phi|[O, H, O^{+}]|\phi\rangle = \omega\langle\phi|[O, O^{+}]|\phi\rangle, \tag{1}$$

where O^{+} is the excitation operator, H is the Hamiltonian of the system and $|\phi\rangle$ is the ground state wave function. The double commutator [a, b, c] is defined as $2[a, b, c] = [a, [b, c]] + [[a, b], c]$. Eq. (1) has been used extensively in nuclear and molecular physics. It is equivalent to the time-dependent Hartree-Fock theory. We applied this method to compute collective-excitation energies of an interacting anyon gas [8]. Correct dispersion relations can be easily obtained. Since calculations of Eq. (1) are straightforward, it can be used easily to investigate the gauge-fluctuation effects on the CF gas. For the excitation operator in the CF problem we construct neutral excitations (particle-hole pairs) from the charge-density operator. We consider the case where the initial Landau levels are completely filled. The elementary neutral excitations of the system are the familiar excitation modes in which one particle is excited to an unoccupied Landau level, leaving behind a hole in a filled Landau level.

Eq. (1) can be written in a non-Hermitian matrix. So that, the usual way to diagonalize a Hermitian matrix cannot be applied to Eq. (1). A simple diagonalization procedure for Eq. (1) is discussed thoroughly by Ullah and Rowe [9]. If we ignore effects from the exchange and 3BGI, we have a RPA solution. The RPA spectra with no CI for $v = 1/3$ are shown in Fig. 1. As expected, at the cyclotron frequency $3\omega_c$ there is a splitting in the dispersion relation for finite q. One of these solutions ω_i has a negative slope and a roton minimum below the cyclotron frequency. Another solution ω_+ has energy above the cyclotron frequency and no roton minimum exists. The mode with *one* ω_c has been "pushed up" to the cyclotron frequency. The mode with $2\omega_c$ has a shallow roton minimum, whereas a deep roton minimum is expected [2, 3]. If we add CI in the RPA solution, collective-energy gaps at $q = 0$ remain the same [5, 6], which indicates that CI have no effect on the incompressibility of the FQHE.

The study, which 3BGI and exchange are included, is an approximation beyond the RPA. Results beyond the RPA are shown in Fig. 1. Collective-excitation energies change dramatically. The 3BGI lift the degeneracy of the cyclotron mode. There is only one cyclotron mode. There is no splitting dispersion relation at finite q. The zero-momentum gap at $3\hbar$

$\hbar$

$\hbar$

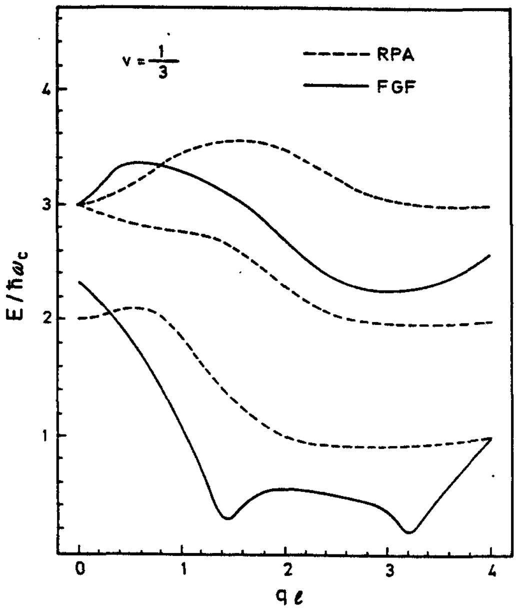

FIG. 1. Excitation energies for $v = 1/3$ without Coulomb interactions. Long-dashed lines are the case of the RPA. Solid lines are the case that full gauge fluctuations (FGF) are considered. The cyclotron energy is $3\hbar\omega_c$. The lowest energy of solid lines is 0.184 $\hbar\omega_c$.

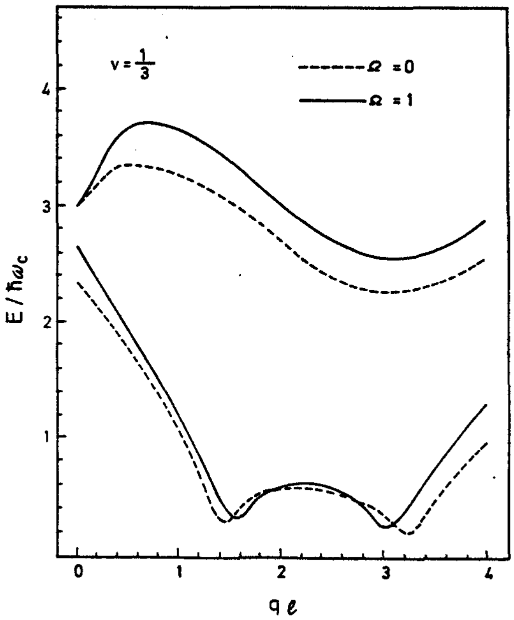

FIG. 2. Excitation energies for $v = 1/3$. Long-dashed lines are the case without Coulomb interactions. Solid lines are results that the strength of Coulomb interactions is equal to 1.

single-exciton work of Kamilla, Wu, and Jain [2]. Here we have the same depth of roton minima from interactions between excitons.

The spectrum of containing CI is shown in Fig. 2. The cyclotron energy at $3\hbar\omega_c$ is unaffected by CI, in accordance with Kohn's theorem. Unlike the RPA, energies of roton minima and the lowest zero-momentum gap are raised by CI. This is consistent with the single-mode approximation [3]. The incompressibility of the CF is stiffer when CI get stronger.

We extend our calculations to other fraction of filling. The dispersion relation for $v = 2/5$ are shown in Fig. 3. Results of including and excluding higher-energy excitons are displayed by (a) diagram and (b) diagram in Fig. 3, respectively. Deep roton minima, analogous to the $v = 1/3$ case, also appear. The relatively complicated structure in the dispersion clarifies why the finite-size calculations are unable to provide a coherent picture. An interesting feature of the dispersion relations is that extra minimum appears at $q\ell < 0.5$ if higher-energy excitons are included in the calculations. There is only a small change of the dispersions at large q whether higher-energy excitons are included or not. Dipolar excitons are responsible for the dispersions at large q. Mixing of higher-energy excitons with low-energy excitons forming a complicated quadrupolar structure is responsible for occurring minima at small q. Quadrupolar excitons can have a lower energy than dipolar excitons at small q [3, 4]. Quasiparticles of 2/5 state have a larger extent than 1/3 state. As q is small and excitons are close, overlapping between excitons to from quadrupolar excitons is larger and easier for 2/5 state than for 1/3 state.

Unlike 1/3 case, 2/5 state has an additional intermediate branch of excitations existing between the lowest branch and the cyclotron-mode branch of spectra. Our calculations indicate that CI have a dramatic effect on the 2/5 state. CI change spectra of 2/5 state not only quantitatively but also qualitatively. In the case of no CI, the energy gap between the intermediate branch and the cyclotron-mode branch of spectra at $q = 0$ is small for 2/5 state. For 2/5 state, energy of the intermediate branch at $q = 0$ can be easily pushed above the cyclotron energy by CI (see Fig. 4). The original intermediate branch then becomes the cyclotron-mode branch. CI raise the lowest energy gap at $q = 0$ and have no effect on the cyclotron energy. Kohn's theorem is also satisfied.

In conclusions we have used the equations-of-motion method to calculate the excitation spectrum of the CF gas. Full gauge fluctuations are considered. 3BGI of the CF model are crucial to the understanding of the FQHE. Effects of CI are revealed clearly if 3BGI are considered explicitly. Except the cyclotron energy. CI raise the gap energies at $q = 0$. CI have no effect on the cyclotron energy, in accordance with Kohn's theorem.

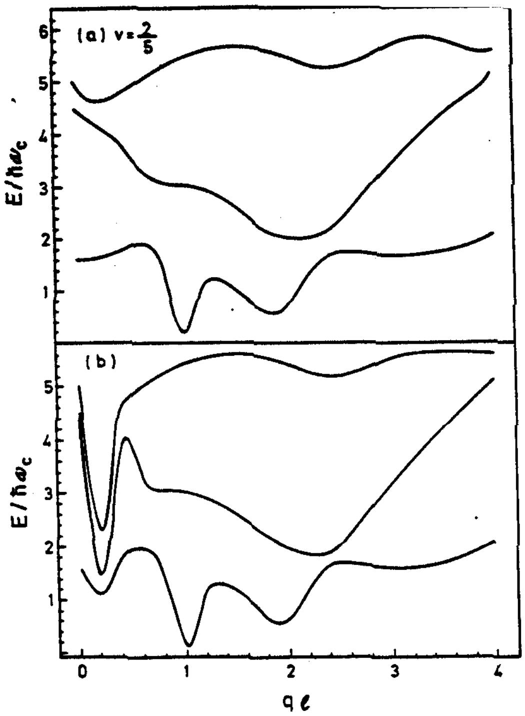

FIG. 3. Excitation energies for $v = 2/5$ without Coulomb interactions. Results from exclusion and inclusion of higher energy excitons in calculations are shown in (a) and (b), respectively. The cyclotron energy is $5\hbar\omega_c$. The lowest energy in (a) is 0.1270 $\hbar\omega_c$.

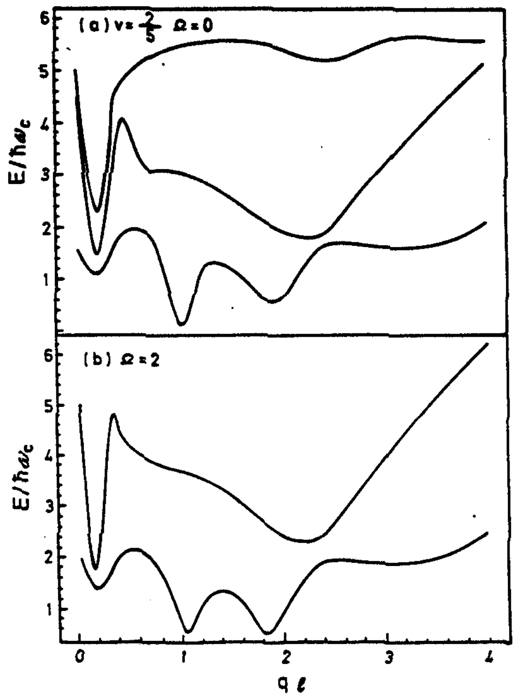

FIG. 4. Excitation energies for $v = 2/5$ from exclusion and inclusion of Coulomb interactions are shown in (a) and (b), respectively. The strength of Coulomb interactions is equal to 2.

This work is supported by the National Science Council of Republic of China under Grant No. NSC86-2112-M-034-001.

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